

Fundamentals on Petroleum and Gas Engineering
(석유가스공학통론) (G17674)

- 2023 Final Examination -

Student ID:

Name:

Notice

- Fill your name in the following:
“I, _____, swear I solve all problems by myself in this final examination. I will take any disadvantages if any dishonesty such as cheating is acted on my solution.”
5 points will be deducted from your total score if you do not fill in your name above.
- **You MUST solve each problem by hand.**
- Submission Deadline: 6:30 PM, June 19, 2023.

Problem 1.

What is the conversion factor between k , expressed in Darcies, and in cm^2 and metre^2 , respectively? [10 pts.]

Problem 2.

2-1. Derive the basic equation for the radial flow of a fluid in a homogeneous porous medium

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{k\rho}{\mu} r \frac{\partial P}{\partial r} \right) = \phi c \rho \frac{\partial P}{\partial t}$$

from

$$q = -\frac{k}{\mu} \frac{\partial \psi}{\partial l},$$

which is the Darcy's law. [5 pts.]

2-2. Derive the radial diffusivity equation. [10 pts.]

2-3. Compare the three most common conditions for which the constant terminal rate solution is sought: transient, semi-steady state, and steady state. [5 pts.]

Problem 3.

A homogeneous formation has an average effective permeability k_e . The effective permeability out to a radius r_a from the well has been altered (damage/stimulation) so that its average value in this region is k_a . Show that, for this situation, the skin factor may be expressed as

$$S = \frac{k_e - k_a}{k_a} \ln \frac{r_a}{r_w}$$

where r_w is the wellbore radius. Assume that for $r < r_a$, the flow can be approximately described under steady state conditions and that for $r > r_a$, semi-steady state. [10 pts.]

Problem 4.

The following reservoir parameters can be evaluated from well testing:

- Initial pressure (p_i)
- Average pressure within the drainage boundary (\bar{p})
- Permeability thickness product (kh), and permeability (k)
- Mechanical skin factor (S)
- Drainage area (A)
- Dietz shape factor (C_A)

Equations 7.10 and 7.13 are the constant terminal rate solution during the transient period and constant terminal rate solution during the semi-steady state period, respectively. It is typical that Equation 7.10 is used to determine k and S for small values of time t while Equation 7.13 is used to determine A and C_A for large values of time t .

The diffusivity equation for oil well testing is as follows:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi \mu c}{k} \frac{\partial p}{\partial t} \quad \text{Equation. (5.20)}$$

From Equation 5.20,

4-1. Show your work to derive Equation 7.10. [5 pts.]

4-2. Show your work to derive Equation 7.13. [5 pts.]

4-3. Show your work how to determine the Dietz shape factor (C_A) using the dimensionless MBH pressure ($p_{D(MBH)}(t_{DA})$). [5 pts.]

Problem 5.

Show your work to solve Exercise 7.6. [5 pts.]

Problem 6.

Show your work to solve Exercise 7.7. [5 pts.]

Problem 7.

The real gas pseudo pressure function $m(p)$ is defined as follows:

$$m(p) = 2 \int_{p_b}^p \frac{p dp}{\mu Z},$$

where p is the pressure, p_b is the base pressure, μ is the gas viscosity, and Z is the gas compressibility factor.

7-1. Derive the radial diffusivity equation for gas well testing, as shown in Equation (8.11). [5 pts].

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial m(p)}{\partial r} \right) = \frac{\phi \mu c}{k} \frac{\partial m(p)}{\partial t} \quad \text{Eq. (8.11)}$$

7-2. Derive the semi-steady state inflow equation for gas well testing, as shown in Equation (8.15). [5 pts].

$$m(\bar{p}) - m(p_{wf}) = \frac{1422QT}{kh} \left(\ln \frac{r_e}{r_w} - \frac{3}{4} + S \right), \quad \text{Eq. (8.15)}$$

where \bar{p} is the average reservoir pressure, p_{wf} is the well-flowing pressure, Q is the flow rate, T is the reservoir temperature, k is the permeability, h is the average reservoir thickness, r_e is the external radius of the reservoir, r_w is the well radius, and S is the skin factor.

7-3. Derive the transient line source solution, when expressed in pseudo pressures and field units, as shown in Equation (8.16). [5 pts].

$$m(p_i) - m(p_{wf}) = \frac{711QT}{kh} \left(\ln \frac{4t_D}{\gamma} + 2S \right), \quad \text{Eq. (8.16)}$$

$$t_D = 0.000264 \frac{kt}{\phi \mu c r_w^2}, \quad \text{Eq. (7.20)}$$

where p_i is the initial reservoir pressure, t_D is the dimensionless time, t is the time, ϕ is the porosity, and c is the compressibility.

Problem 8.

Datasets 1–3 summarize available rock and fluid properties and well data for a newly discovered gas reservoir, called EWHA Gas Field, which is assumed as isothermal and homogeneous before building a full-physics reservoir model. Those data have been measured through volumetric analysis (Dataset 1), lab-scale PVT test (Dataset 2), and field-scale well test (Dataset 3) at an exploration well. Of course, all data are associated with intrinsic reservoir uncertainty.

Although a gas sample was collected during a brief production test (See Dataset 3), the initial reservoir pressure was not recorded because of tool failure. It is known, however, that the water pressure regime in the locality is

$$p_w = 0.44D + 24 \text{ psia,}$$

where p_w is the hydrostatic pressure and D is the depth (ft).

(Tip: You can refer to Exercise 1.2 for solving the problem.)

- 8-1. Calculate the gas pressure at the centroid. Regard this pressure as the initial pressure of the gas-bearing formation (i.e., gas reservoir). [4 pts.]
- 8-2. Calculate the reservoir temperature at the centroid. Regard this temperature as the reservoir temperature under the assumption the gas reservoir is isothermal. [4 pts.]
- 8-3. Calculate the average reservoir thickness h , where $h = 2(\text{GWC} - \text{depth at the centroid})$. [2 pts.]
- 8-4. Calculate the gas expansion factor E at the centroid, and regard this E value as the representative E value for the subsequent well testing problem. [4 pts.]
- 8-5. Calculate the pore volume (PV) of the gas reservoir. [2 pts.]
- 8-6. Calculate the volume of the GIIP (Gas Initially in Place). [4 pts.]

Dataset 1. Reservoir and fluid properties and well data

Parameter	Value	Unit
GWC	9,700	ft
Centroid depth	9,680	ft
Net bulk volume (V)	1.7760E+10	ft ³
Porosity (ϕ)	0.20	fraction
Connate water saturation (S_{wc})	0.20	fraction
Gas gravity (γ or γ_g)	0.85	fraction
Equation of water pressure gradient (i.e., hydrostatic pressure gradient)	0.44 * D (Depth) + 24 psia	
Temperature gradient (i.e., geothermal gradient)	0.0124	°F/ft
Ambient surface pressure	14.7	psia
Ambient surface temperature	80	°F
Product of initial viscosity and compressibility (μc) _i	3.6E-6	cp/psi
Well radius (r_w)	0.3	ft
Geometry	Radial flow from a bounded reservoir (See Fig. 6.4)	

Dataset 2. PVT data

- Note that this table is the same to Table 8.1.

PVT Data		
p	μ	Z
400	0.01286	0.937
800	0.0139	0.882
1200	0.0153	0.832
1600	0.0168	0.794
2000	0.0184	0.77
2400	0.0201	0.763
2800	0.0217	0.775
3200	0.0234	0.797
3600	0.025	0.827
4000	0.0266	0.86
4400	0.02831	0.896

Dataset 3. Well test data

Below is the table that contains time-series gas well testing data, composed as follows:

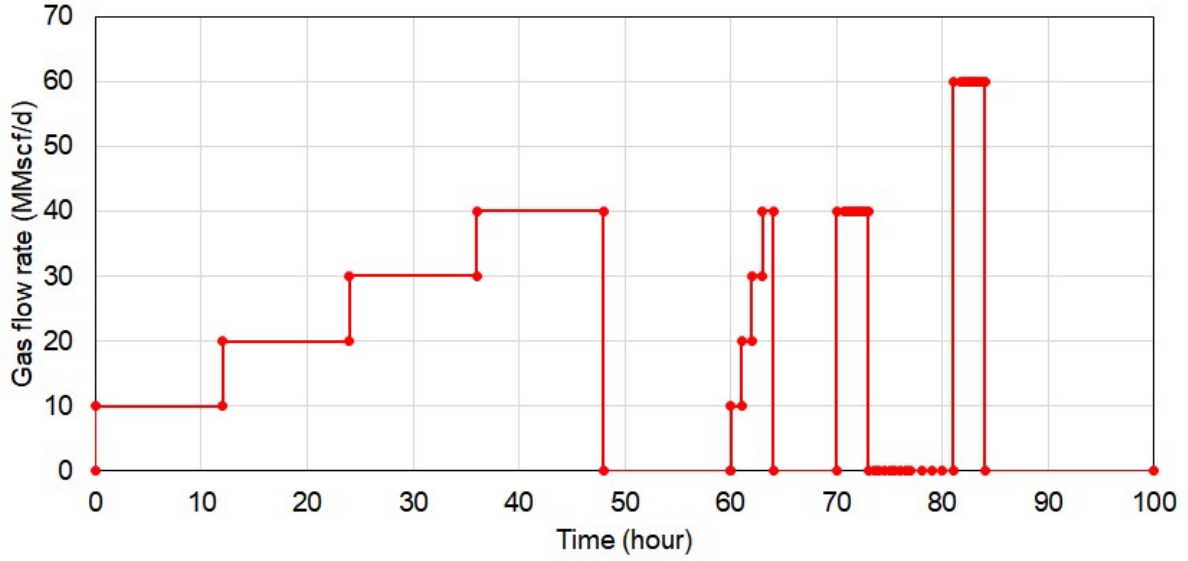
- Well Test #1: Multi-rate drawdown test with four different flow rates of 10, 20, 30, and 40 MMscf/d for periods of 12 hours, followed by a 12-hour pressure buildup (Note that the Well Test #1 is complete at $t = 48$ hours).
- Well Test #2: Multi-rate drawdown test with four different flow rates of 10, 20, 30, and 40 MMscf/d for periods of 1 hour, followed by a 6-hour pressure buildup.
- Well Test #3: Pressure-buildup test with a flow rate of 40 MMscf/d for 3 hours, shut-in for 8 hours, and a flow rate of 60 MMscf/d for 3 hours, and well shut-in, in sequence. Note that bottomhole static pressure P_{ws} data are measured only during Well Test #3 (pressure buildup).

Time hour	Flow Rate (Q) MMscf/d	BHP (P_{wf} or P_{ws}) psia
0.00	0	
0.00	10	
12.00	10	4182
12.00	20	
24.00	20	4047
24.00	30	
36.00	30	3885
36.00	40	
48.00	40	3694
48.00	0	
60.00	0	
60.00	10	
61.00	10	4160
61.00	20	
62.00	20	3998
62.00	30	
63.00	30	3800
63.00	40	
64.00	40	3580
64.00	0	
70.00	0	
70.00	40	
70.75	40	3600
71.00	40	3595
71.25	40	3590
71.50	40	3587

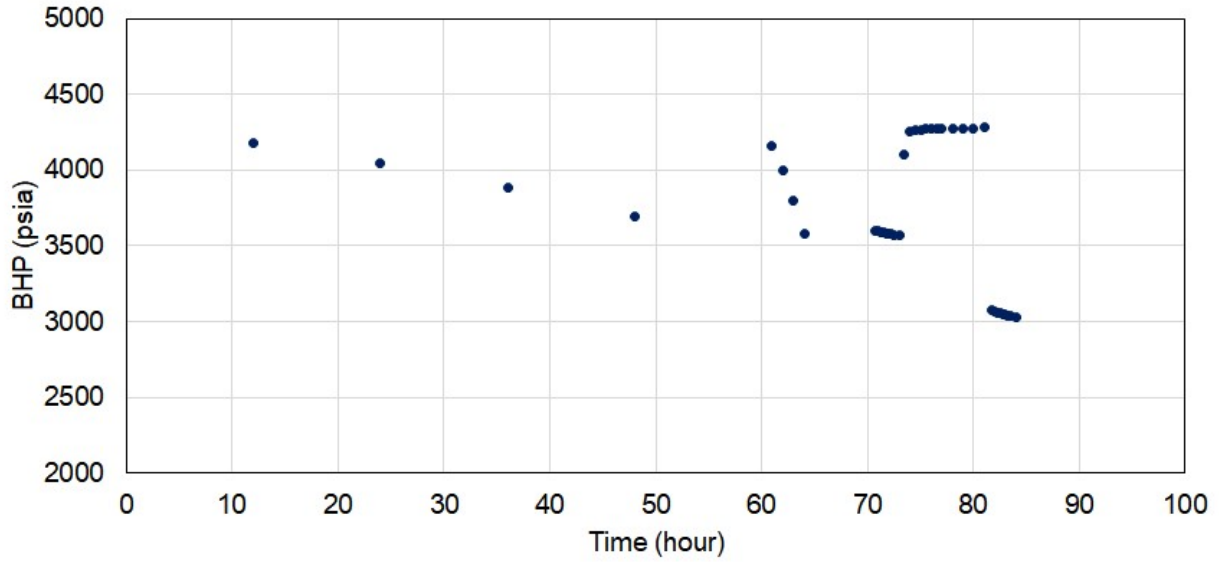
71.75	40	3583
72.00	40	3580
72.25	40	3577
72.50	40	3575
72.75	40	
73.00	40	3570
73.00	0	
73.50	0	4100
74.00	0	4255
74.50	0	4263
75.00	0	4267
75.50	0	4269
76.00	0	4271
76.50	0	4273
77.00	0	4274
78.00	0	4276
79.00	0	4277
80.00	0	4278
81.00	0	4279
81.00	60	
81.75	60	3075
82.00	60	3065
82.25	60	3058
82.50	60	3053
82.75	60	3047
83.00	60	3044
83.25	60	3039
83.50	60	3035
83.75	60	
84.00	60	3032
84.00	0	
100.00	0	

Below are figures to show gas flow rate and BHP (Bottomhole Pressure) (P_{wf} or P_{ws}) drawn using the dataset 3.

Well Test Data from an EWHA Gas Field



Well Test Data from an EWHA Gas Field



----- This is the End of the Final Examination -----