Spatial Information Modeling for Climate and Energy Systems
(기후에너지 공간정보모델링) (38541)

- 2023 Final Examination -

Student ID: **Name:**

Notice

- Fill your name below and write the whole sentence in your answer sheet:
  "I, ________________, swear I solve all problems by myself in this final examination.
  I will take any disadvantages if any dishonesty such as cheating is acted on my solution."
  5 points will be deducted from your total score if you do not fill in your name above.
- **You MUST solve each problem by hand.**
- Submission Deadline: 12:30~13:45, June 8, 2023.

Problem 1. [10 pts.]

For each of sub-problems 1-1 to 1-5, which answer is correct? Choose either “Same” or “Can be different”.

	Geometric model (기하모델)	Zonal model (구역모델)
Variogram models along the major and the directions	Example: Same vs. Can be different	1-1. Same vs. Can be different
Sill	1-2. Same vs. Can be different	1-3. Same vs. Can be different
Range	1-4. Same vs. Can be different	1-5. Same vs. Can be different

Problem 2. [20 pts.]

Draw five theoretical variogram models (i.e., nugget, linear, spherical, exponential, and Gaussian models) with their formulae as a function of distance h with a unit range (a = 1) and a unit sill (σ^2 = 1) in a single graph. Compare characteristics of these variogram models near the origin and at the range of a = 1.

Problem 3. [5 pts.]

In the Cartesian coordinate system, calculate the semi-variogram at (x, y) = (3, 4) when the anisotropic semi-variogram model of which the major direction is N30°E is as follows:

$\gamma_x(h) = 2 + 3\text{Gauss}10_{10}(h)$

$\gamma_y(h) = 2 + 3\text{Gauss}10_{5}(h)$

Note that distance h must be calculated from the origin (x, y) = (0, 0).

Problem 4. [20 pts.]

Estimate a spatial random variable z at any location using n sample data points. Here, the estimate is denoted as $z^* = m + \sum_{i=1}^{n} \lambda_i (z_i - m)$, where m is the population mean.

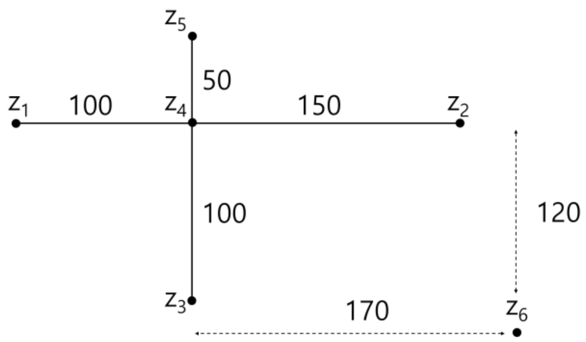
4-1. Show your work to derive the Kriging equation and error variance for Simple Kriging (SK) [10 pts.].

4-2. Show your work to derive the Kriging equation and error variance for Ordinary Kriging (OK) [10 pts.].

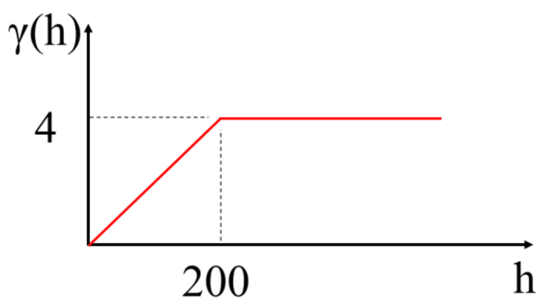
Problem 5. [20 pts.]

Estimate kriged values and its error variance values at z_4, z_5, and z_6 using Ordinary Kriging (OK) under the following conditions:

- Variogram model is linear with the range of 200 and sill of 4 (i.e., $\gamma(h) = 4\text{Linear}_{200}(h)$).
- Three sample values are as follows: $z_1 = 5$, $z_2 = 10$, and $z_3 = 15$.
- Round any number to the first decimal place (소수 첫째자리까지) for your own calculation.
- CAUTION: For each z estimate, you MUST show your Kriging Equation in a matrix form. Every element in the matrix MUST be written to the second decimal place.



(a) Distribution of sample data z_1, z_2, and z_3



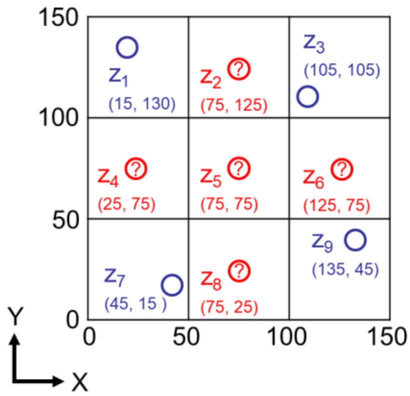
(b) Variogram model (linear)

Problem 6. [25 pts.]

In a two-dimensional domain, X and Y are coordinates and Z is the content of gold in rock sample. The unit of Z is gold karat (g/ton). Four rock samples are collected from Z_1, Z_3, Z_7, and Z_9. Make a rational assumption, if needed.

- CAUTION: For each z estimate, you MUST show your Kriging Equation in a matrix form. Every element in the matrix MUST be written to the second decimal place.

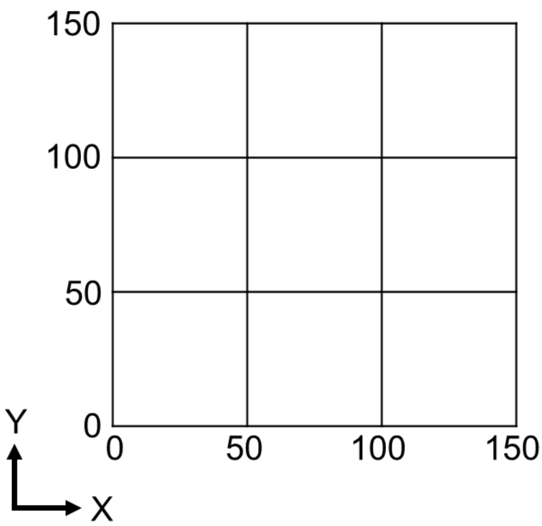
Variogram model
: $\gamma(h) = 1 + 3\mathrm{Exp}_{120}(h)$



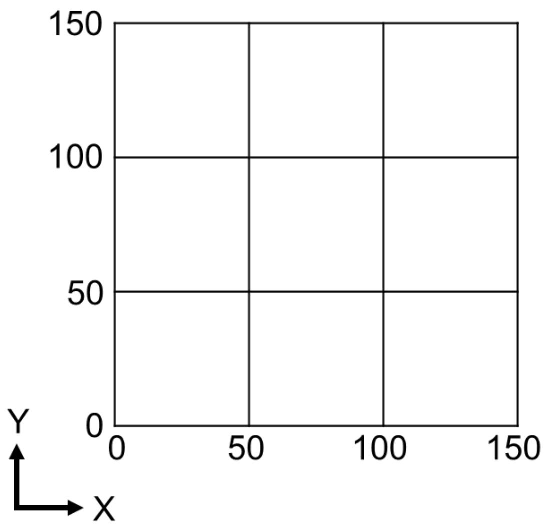
Data No.	X	Y	Z, g/ton
1	15	130	8
2	75	125	?
3	105	105	9
4	25	75	?
5	75	75	?
6	125	75	?
7	135	45	12
8	75	25	?
9	45	15	10

6-1. Show your work to draw a map of Z estimates using Ordinary Kriging (OK), in detail. In other words, show your work how to estimate Z values from Z_1 to Z_9, in detail.

6-2. Show your work to draw a map of error variance σ^2_{OK} Ordinary Kriging (OK), in detail. In other words, show your work how to estimate error variance associated with Z values from Z_1 to Z_9.



6-1. Gold content map
from ordinary kriging



6-2. Error variance map
from ordinary kriging

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