# Spatial Information Modeling for Climate and Energy Systems (기후에너지 공간정보모델링) (38541)

## - 2024 Final Examination -

**Student ID:** 

Name:

### Notice

• Fill your name below and write the whole sentence in your answer sheet:

"I, \_\_\_\_\_, swear I solve all problems by myself in this final examination. I will take any disadvantages if any dishonesty such as cheating is acted on my solution."

#### 5 points will be deducted from your total score if you do not fill in your name above.

- You MUST solve each problem by hand.
- Submission Deadline: 11:00~12:15, June 10, 2024.

#### Problem 1. [20 pts.]

Draw five theoretical variogram models (i.e., nugget, linear, spherical, exponential, and Gaussian models) with their formulae as a function of distance *h* with a unit range (a = 1) and a unit sill ( $\sigma^2 = 1$ ) in a single graph. Compare characteristics of these variogram models near the origin and at the range of a = 1.

#### Problem 2. [20 pts.]

For Universal Kriging (UK), let us denote z(x) = D(x) + R(x), where D(x) is a drift term (i.e., trend) and R(x) is a residual term.

- 2-1. Derive Kriging equation in a matrix form.
- 2-2. Derive error variance in a general form.

#### Problem 3. [20 pts.]

Estimate kriged values and its error variance values at  $z_4$ ,  $z_5$ , and  $z_6$  using Ordinary Kriging (OK) under the following conditions:

- Variogram model is linear with the range of 200 and sill of 4 (i.e.,  $\gamma(h) = 4$ Linear<sub>200</sub>(h)).
- Three sample values are as follows:  $z_1 = 5$ ,  $z_2 = 10$ , and  $z_3 = 15$ .
- Round any number to the first decimal place (소수 첫째자리까지) for your own calculation.
- CAUTION: For each z estimate, you MUST show your Kriging Equation in a matrix form. Every element in the matrix MUST be written to the second decimal place.



(a) Distribution of sample data  $z_1$ ,  $z_2$ , and  $z_3$ 

(b) Variogram model (linear)

#### Problem 4. [40 pts.]

In a two-dimensional domain, X and Y are coordinates and Z is the content of gold in rock sample. The unit of Z is gold karat (g/ton). Four rock samples are collected from  $z_1$ ,  $z_2$ ,  $z_3$ , and  $z_4$ . You may make rational assumptions, if necessary.

Variogram model :  $\gamma(h) = 1 + 3 \operatorname{Sph}_{120}(h)$ 



Data No.	Х	Y	Z, g/ton
1	15	130	8
2	105	105	9
3	135	45	10
4	45	15	12
0	75	75	?

• CAUTION: For each z estimate, you MUST show your Kriging Equation in a matrix form. Every element in the matrix MUST be written to the third decimal place.

6-1. Show your work to estimate  $z_0^*$  and its error variance  $\sigma_{SK}^2$  using Simple Kriging (SK), in detail.

6-2. Show your work to estimate  $z_0^*$  and its error variance  $\sigma_{OK}^2$  using Ordinary Kriging (OK), in detail.

6-3. Show your work to estimate  $z_0^*$  and its error variance  $\sigma_{BK}^2$  using Block Kriging (BK), in detail. You may select

6-4. Show your work to estimate  $z_0^*$  and its error variance  $\sigma_{CK}^2$  using Co-Kriging (CK), in detail. Here, the secondary variable  $u_3 = 135$ ,  $\gamma_u(h) = 5 + 200Exp_{100}(h)$ ,  $\gamma_{zu}(h) = 5 + 200Exp_{100}$ .

----- This is the End of the Final Examination ------